An Experimental Analysis of Private Information in Constrained Contracting*

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Abstract

I experimentally test a model of how organizations write rules when control over the action is limited and there is asymmetric information. A principal (writer) writes a rule that dictates an action for an agent (receiver) to take. The action is based on the information that the principal receives. The principal has an incentive to shape how the information is used, but has limited control because of the complexity both of the information they receive and of describing the action. If a principal does not retain control over the action for some information they may receive, the principal privately observes the information and has a chance to communicate to the agent. The principal has the core problem of how to optimally exercise limited control. In addition, the form of limited control may impact communication. This paper experimentally tests how rules are written and how complete rules may be. It is predicted that as preferences become more aligned between the principal and the agent, there is more scope for communication. Experimentally, this prediction is supported. Many subjects choose not to write a contract, even though it is theoretically optimal to do so. It is hypothesized that this is due to communication providing high average payoffs after no contract has been written. Subjects fail to write rules that divide the information into two different categories to facilitate clearer communication.

JEL: C90, D23, D71, D86, K12
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1 Introduction

Heads of organizations have a complex and challenging task at hand when deciding upon rules (contracts) that govern the actions that must be taken. A principal who writes a rule that determines the action for an agent to take must decide the appropriate events to include in the contract, which actions to take in cases of those events, and must carefully describe each event and action. In organizations, the information that a principal may observe and the actions that an agent may need to take can be complex to describe. The principal must then carefully decide the amount of detail to include in the rule, since it is impossible for the principal to write a completely detailed contract. Furthermore, the decision of what to include depends on the nature of what happens outside of clauses in the rule. Because only the principal observes all relevant information, states that fall outside of the rule are subject to communication between the principal and the agent. The principal must deduce how to optimally exercise limited control when the alternative to their control is communicating to the agent.

This experiment tests how writers write rules when they face this problem of limited control. In the experiment, a writer will, prior to observing the state of the world, write a contract. A "contract" will indicate three things: a low state, a high state, and a writer action. The contract stipulates that if a state is drawn between the low state and the high state (inclusive), the writer action will be taken. After the contract is written, a state is randomly drawn. If the state is lower than the low state or higher than the high state, the writer will privately observe the state. After observing the state, the writer will send a message to the receiver (acting as an interpreting party), who observes only the message sent and the contract written. If the receiver receives a message, the receiver will take a receiver action and the game ends. Because the receiver's ideal action given the state is different than the writer's ideal action given the state, there is a conflict of interest between the two players.

The writer has a difficult optimization problem: The writer can write a contract that fixes the action for more states, or the writer can rely more heavily on communication. Additionally, the writer can write a contract to induce different types of communication. At one extreme, the writer can write no contract and rely only on communication. After no contract has been written, the game looks similar to the model of communication presented in Crawford & Sobel (1982) . At the other extreme, the writer can write a complete contract that stipulates a single action to be taken in every state of the world. This makes the action deterministic, meaning that the action has no sensitivity to the state. For no conflict of interest between the writer and receiver, only relying on communication is optimal, while for a large enough conflict of interest, a complete contract is optimal. For intermediate conflicts of interest, it is optimal for the writer to write partially complete contracts. A treatment with low conflict of interest and a treatment with high conflict of interest are utilized to test whether the completeness of a contract is sensitive to the preferences of the receiver. Experimental evidence supports that subjects write more complete contracts when their interests are less aligned.

However, subjects do not behave in accordance with other predictions. Given the states included in the contract, writers tend to include actions that are far from optimal. Writers also tended to write contracts that covered too many states in the low-conflict treatment and too few states in the high-conflict treatment. It is predicted that contracts should be written such that the writer can clearly communicate when a state is above or below the contracted region, but over half of the time, contracts are not written in this fashion. 30.2% of writers in the low bias treatment and 26.7% of writers in the high bias treatment chose to not write a contract and instead rely solely on communication. According to predictions, writers should always choose to write a contract.

One critical reason that some writers utilized full communication involves what happens when players reach the communication stage: payoffs are higher than predicted when the writer chooses not to write a contract. This is primarily due to messages being overly indicative of the state. On the other hand, there is undercommunication when the contract splits the remaining states into two separate regions, which leads to poor outcomes for both players. In addition, there was a small learning effect in both treatments as the number of states covered in the contract shrinks over time. I postulate that this is due to the poor performance of contracts with many states. However, there is not a clear causal link between the interpretation rule and the writing of the contract. It is unclear whether communication had an impact on the writing of contracts, or whether the specific writing of the contracts impacted communication. This remains to be the subject of future work.

In addition to the sessions in the US, I also ran experiments in Japan at Waseda University. I find that the play in the Japanese sessions more closely matched theory: errors in writer actions were smaller in magnitude and fewer, and the number of states covered in the contract was more different between treatments when compared to the Arizona group. However, the number of states included was similar in each treatment between Arizona and Japan. Many people (although fewer in the high bias Japan sessions) also chose not to write contracts. I postulate that some of these differences have to do with the high difference in sophistication of the subject pool. Because writing an action in a contract is a complex math problem, is it unsurprising that a subject pool with higher average CRT scores performed better.

The theoretical framework of the experimental model is examined in detail in Blume, Deimen & Inoue (Working Paper). The setting is generalized to an interval of states, a general utility function, and a fixed number of terms allowed in the contract. We find that decreasing the bias leads to more communication, while increasing the allowed number of terms in the contract decreases the amount of communication. Additionally, the contract serves to facilitate communication. I find experimental evidence that the amonut of control is responsive to the bias, but find that the contract can harm communication between subjects.

There is experimental work that analyzes how subjects communicate. Cai & Wang (2006) experimentally test the results of the theoretical framework of Crawford & Sobel (1982). That experiment shows that senders (writers) tend to overcommunicate information and that receivers tend to believe said information is true. In addition, payoffs tend to be close to what is predicted in the theory of Crawford & Sobel. This overcommunication result is confirmed in Wang et. al. (2010). In this paper, analyzing communication subgames after a contract has been written shows that there are both communication subgames in which payoffs were better than predicted and communication subgames in which payoffs were worse than predicted. This paper adds to the literature by analyzing communication with different restrictions on the states. Unlike the result in Cai & Wang that shows only overcommunication, this paper shows that subjects overcommunicate in some communication subgames and undercommunicate in others. This result of having settings in which undercommunication occurs is additionally highlighted in Blume et. al. (2001), who in one treatment (Game 4) find that one type sends a message that pools with another type without an incentive to do so.

There is a large body of theoretical work focused on incomplete contracts with restrictions on the complexity of a contract. In these settings, it is sometimes optimal to intentionally write incomplete contracts. Dye (1982) is one of the first models in this literature, detailing how contractual incompleteness can come about in markets. Simon (1951) identifies how contractual incompleteness arises in employer/employee relationships. Shavell (2006) analyzes the role of interpretation in contracts, and solves for which interpretive rules are optimal in an incomplete contracting framework. Heller & Spiegler (2008) add to this framework by allowing for contradictory statements to exist in a contract. In these detailed theoretical models, the authors are interested in which contracts written by the writers maximize the writers' payoff in equilibrium, as well as the optimal interpretive rule chosen by the party interpreting gaps or contradictions. In each of these papers, there are common themes regarding the way in which the contract is written. For example, as conflict of interest increases, those papers and that model predict contracts to be more complete. This paper experimentally tests whether human subjects write contracts in accordance with those common themes.

Previous work has explored how varying the setting impacts how agents write contracts. Fehr & Schmidt (2007) analyze how fairness impacts contract design. Brandts, Charness, & Ellman (2012) analyze how communication affects the design of a contract, which is a key question of this paper. However, the setting of this paper focuses on communication after a contract has been written as an interpretive rule instead of focusing on the impact of ex-ante, free-form communication in forming agreements.

The remainder of the paper will proceed as follows: Section two provides an overview of the experimental setup and characterizes the set of optimal contracts, as well as describing the set of predictions tested in the experiment. Section three discusses the experimental design. Section four analyzes the experimental results and the sessions with subjects at Waseda University in Japan. Section five concludes the paper.

2 Experimental Setup

2.1 Description

There are two agents: a writer and a receiver. In the game G, a writer will be writing a contract (called a "rule" in the experiment) that dictates an action to be taken in certain states, while the receiver will be providing interpretations for messages the writer sends when the state is outside of the contract. The writer has payoff $U_W(s, a; b) = 30 - |s + b - a|^{1.4}$, while the receiver has payoff $U_R(s, a) = 30 - |s - a|^{1.4}$, where $s \in S = \{1, 3, 5, 7, 9\}$ is a state drawn from a uniform distribution over the state space S, $a \in \{\mathbb{R} \bmod 0.25\}$ is the action taken, and $b \geq 1$ is the bias term.[1] Note that, given the state is common knowledge, if the action were continuous, the optimal writer action would be $a_W^* = s + b$ while the optimal receiver action is $a_R^* = s$.

In stage one, the writer writes a contract (rule) $C = (s_{low}, s_{high}, a_W)$ that indicates a low state s_{low}, a high state s_{high}, and a writer action a_W. Informally, the contract states that when a s is drawn that is between the low state and the high state or equal to either of the two states, the writer action is taken. The writer is allowed to write no contract, in which case $C = \emptyset$.

[1] $b \geq 1$ is used instead of $b \geq 0$ because this eliminates almost all possible mixed strategy equilibria. These equilibria arise due to the discreteness of the state space, so using $b \geq 1$ simplifies the analysis significantly while preserving the application of the analysis to the experiment

In stage two, the state $s \in S = \{1, 3, 5, 7, 9\}$ is drawn from a uniform distribution over S. If $s_{low} \leq s \leq s_{high}$, then the action $a = a_W$ is taken and the game terminates. If $s < s_{low}$ or $s > s_{high}$, the writer privately observes s and the game proceeds to stage 3.

In stage three, if the game has not ended, the writer sends a costless message $m \in \{0, 1, ..., 9, 10\}$ to the receiver.[2]

In stage four, the receiver observes the message and then takes an action a_R, which causes the game to end. Actions are allowed to be multiples of 0.25. When the game ends, the writer and receiver both realize payoffs and observe the state, the action taken, and the contract.

To introduce some terminology, a *gap* is be defined as the set of states not covered by the contract, $\{s < s_{low}\} \cup \{s > s_{high}\} = \mathcal{G}$. A *lacuna* is defined as either $\{s < s_{low}\}$ or $\{s > s_{high}\}$. A contract is considered *obligationally complete* if there is no gap ($C = (1, 9, a_W)$) and otherwise is considered incomplete.

A communication subgame, Γ^C, is defined as a game in which a state is drawn from a uniform distribution over $S^C = \{s \notin \{s_{low}, \ldots, s_{high}\}\} \subseteq S$ that the writer privately observes. The writer then sends a message $m \in M$ to the receiver. After the receiver receives a message, they take an action a_R. Note that Γ^C only exists if $C \neq (1, 9, a_W)$. At Γ^C, a strategy for the writer $\sigma_W^C : \mathcal{G} \rightarrow \Delta(M)$ maps from the gap into distributions over the message space. A strategy for the receiver $\sigma_R^C : M \rightarrow \Delta(\mathbb{R})$ maps from the messages into distributions over the action space. In the overall game G, a strategy for the writer $\left(C, \left(\sigma_W^{C'}\right)_{C' \in \mathbf{C}}\right)$ is a contract and a strategy for the writer within each communication subgame. A strategy for the receiver $\left(\sigma_R^{C'}\right)_{C' \in \mathbf{C}}$ is a strategy for the receiver within each communication subgame. This paper is concerned with optimal contracts, where an optimal contract is defined as the contract that yields the highest payoff to the writer in any pure-strategy perfect Bayesian equilibrium. The next section will formally characterize optimal contracts.

2.2 Observations

This section outlines properties of optimal contracts. These properties will be utilized to make predictions about the experimental results. The experimental test will use two treatments: $b = 1.25$ and $b = 2.25$. In each of these treatments, contracts are predicted to be incomplete in different ways. These two treatments will be used to test the primary predictions of the model. This section begin with some observations that will narrow the range of possible equilibria. Next, the section will give a full characterization of optimal contracts

[2] This includes messages 0 and 10 in case the writer wants to exaggerate about the state.

for all $b \geq 1$. Following this will be notes on properties that will be of interest when making predictions about play in the following section.

The first observation is a characterization of the best possible writer action. It additionally characterizes the best possible receiver action if the receiver knows the states over which a message is sent. The action is pinned down by the structure of the payoffs combined with the uniformly distributed states.

Observation 1 *Given a low state s_{low} and a high state s_{high}, the optimal contract specifies the writer action $a_w = \frac{s_{low}+s_{high}}{2}+b$. Conditional on the receiver knowing the state $s \in S' \subseteq S$, the optimal receiver action is $\mathbb{E}[s \mid s \in S']$.*

It is also possible to write down the structure of perfect Bayesian equilibria in any communication subgame. After all possible equilibria are found, optimal contracts will be found by finding sender-optimal equilibria and comparing contracts given that the equilibrium is sender optimal.

Observation 2 *In any communication subgame, a perfect Bayesian equilibrium of that subgame is represented as a partition $P = \{p_1, \ldots, p_n\}$ for $n \geq 1$, where $p_i = \left\{s_1^i, ..., s_{k_i}^i\right\}$ is a partition element. Any message $m \in M_i \subseteq M$ that is sent for $s^i \in p_i$ induces a unique expected action $a_R^i \neq a_R^j \forall j$, where a_R^i is the expected receiver preferred action given $m \in M_i$ is sent. Each partition element is a convex set in S and ordered such that for any $s^i \in p_i$, $s^j \in p^j$, $s^i < s^j$.* [3]

Observation 3 *A writer will always choose to write a contract.*

The key is that, in any communication equilibrium, getting rid of the lowest partition element does not change the remainder of the states being in equilibrium. If a contract is inserted where the lowest partition element is, a strict payoff improvement can be gained by the writer, who now gets their preferred action in that region of the state space.

The next two observations fully outline a characterization of the optimal contract. The optimal contract for any b will be shown in table 1. The details of this calculation will be shown in Appendix C.

Observation 4 *The optimal contract for any $b > 1$ is found by first computing all writer-optimal perfect Bayesian equilibria within each communication subgame. Given that any communication subgame will contain an equilibrium that maximizes the writers payoff, the optimal contract is the contract that selects a communication subgame in a way that maximizes the writer's payoff. The optimal contract for all $b > 1$ is displayed in figure 1.*

[3] Using the weak topology over S

Observation 5 *The number of states specified in the optimal contract weakly decreases as the bias decreases.*

Table 1: Optimal Contracts

Range of b	Optimal Contract(s) (s_{low}, s_{high}, a_W)	Expected Payoff to Writer	Expected Payoff to Receiver
$b = 1$	$(s, s, s+b)\,\forall s \in \{1,3,5,7,9\}$	$30 - \frac{4\lvert b\rvert^{1.4}}{5}$	$30 - \frac{\lvert b\rvert^{1.4}}{5}$
$1 < b \leq 1.5$	$(3,3,3+b)$,$(7,7,7+b)$	$30 - \frac{2\lvert b\rvert^{1.4}+\lvert b-1\rvert^{1.4}+\lvert b+1\rvert^{1.4}}{5}$	$30 - \frac{\lvert b\rvert^{1.4}+2}{5}$
$1.5 < b \leq 2.149$	$(3,5,4+b)$,$(5,7,6+b)$	$30 - \frac{2+\lvert b\rvert^{1.4}+\lvert b-1\rvert^{1.4}+\lvert b+1\rvert^{1.4}}{5}$	$30 - \frac{\lvert b+1\rvert^{1.4}+\lvert b-1\rvert^{1.4}+2}{5}$
$2.149 < b \leq 3.610$	$(3,7,5+b)$	$30 - \frac{2\lvert 2\rvert^{1.4}+2\lvert b\rvert^{1.4}}{5}$	$30 - \frac{\lvert b+2\rvert^{1.4}+\lvert b\rvert^{1.4}+\lvert b-2\rvert^{1.4}}{5}$
$3.610 < b \leq 4.375$	$(1,7,4+b)$,$(3,9,6+b)$	$30 - \frac{2\lvert 3\rvert^{1.4}+2+\lvert b\rvert^{1.4}}{5}$	$30 - \frac{\lvert 3+b\rvert^{1.4}+\lvert 1+b\rvert^{1.4}+\lvert 1-b\rvert^{1.4}+\lvert 3-b\rvert^{1.4}}{5}$
$b \geq 4.375$	$(1,9,5+b)$	26.159	$30 - \frac{\lvert 4+b\rvert^{1.4}+\lvert 2+b\rvert^{1.4}+\lvert b\rvert^{1.4}+\lvert 2-b\rvert^{1.4}+\lvert 4-b\rvert^{1.4}}{5}$

2.3 Predictions

1. Writers will always write contracts.
2. Each gap, if it exists, will be such that communication is utilized on either side of the contract. That is, $s_{low} > 1$ and $s_{high} < 9$.
3. The contract that the writer writes is such that $a_W = \frac{s_{low}+s_{high}}{2} + b$.
4. As b increases from 1.25 to 2.25, the number of states covered by a contract will increase.
5. In any communication subgame, communication will be consistent with the most informative perfect Bayesian Nash equilibrium.
6. Writers are more likely to switch contracts after receiving a poor payoff.

While predictions 1-4 have to do with the contract writing, prediction 5 analyzes behavior within communication subgames. In an equilibrium, given contracts written by writers, communication should be optimal. Given behavior within communication subgames, writers will choose the contract that gives them the best payoff. If prediction 5 is violated, communication is not in equilibrium. If prediction 6 holds, then writers are sensitive to negative payoff shocks and are attempting to select contracts based on the payoffs they earn.

3 Experimental Design

Subjects completed the task at the Experimental Science Laboratory at the University of Arizona. The experiment was coded in z-Tree. Subjects participated in a total of eight sessions in the US–four for each treatment. The $b = 1.25$ treatment had a total of 54 subjects–27 writers and 27 receivers. The $b = 2.25$ treatment had a total of 58 subjects–29 writers and 29 receivers. Each session had between 8 and 18 people. Within each session, each subject played two practice rounds of the game. In each practice round, subjects played by themselves and made the decisions of both roles. At the end of those practice rounds, each subject was quizzed on the results. These quizzes had no payout implications, but subjects were encouraged to ask questions if they got the quiz wrong. Following the quiz phase, subjects played 30 rounds, with roles fixed as either the writer or the receiver across all rounds. Matching was done randomly. At the end of the experiment, subjects were paid for two of the thirty rounds chosen randomly. Each subject earned 33 cents per ECU, along with a show-up fee of $10. The experiment took between 105 minutes and 150 minutes. Subjects were paid an average of $27.27. The only difference between the setup above and the experiment is that subjects were restricted to actions that were a multiple of 0.25 that were between 1 and 12.

In addition, as a robustness check, instructions and the z-Tree file were translated into Japanese by a Waseda University graduate student. Subjects participated in two sessions at the Experimental Science Laboratory at Waseda University in Japan with the help of Waseda faculty and graduate students. There was one session for each treatment. The $b = 1.25$ treatment had 22 total subjects–11 in each role–and the $b = 2.25$ treatment had 20 total subjects, with 10 in each role. Each subject was paid 37 yen per ECU, along with a 1000 yen show up fee. Each session took two hours. Subjects were paid an average of 2995 yen.

4 Experimental Results

4.1 How People Write Contracts

This section discusses whether subjects played in accordance to predictions 1-4. These predictions focus on how writers write contracts without detailed analysis on how communication might impact results. A summary of the total number of contracts written of each type is presented in table 2

Result 1. Prediction one is supported. Across both treatments, subjects commonly chose to not write contracts.

For result one, table 3 shows the fraction of periods in which no contract was written, while table 4 shows the fraction of subjects who chose not to write a contract in at least 10/20 periods. Over all, roughly 30% of periods in the $b = 1.25$ treatment and roughly 25% of periods in the $b = 2.25$ treatment had no contract written. Over time, the number of subjects abstaining from writing a contract increases. In addition, looking at subject-level data, roughly a third of subjects wrote no contract a third of the time in each treatment, while roughly a quarter of subjects wrote no contract two-thirds of the time in each treatment. Given that there are a significant portion of subjects choosing to not write contracts, and given that learning seems to go in the opposite direction of the prediction, prediction one can be rejected.

Result 2. Prediction two is not supported. Across both treatments, subjects commonly choose to write a contract that included either $s_{low} = 1$ or $s_{high} = 9$.

Table 2: Events that Writers Wrote in Contracts

(Low State, High State)	b=1.25		b=2.25	
	Total Instances	Number of Unique Subjects	Total Instances	Number of Unique Subjects
No Contract	245	16	232	14
(1,1)	25	4	12	4
(3,3)	19	6	31	6
(5,5)	21	6	37	8
(7,7)	5	4	15	4
(9,9)	34	5	48	7
(1,3)	94	13	37	7
(3,5)	72	11	20	8
(5,7)	18	9	39	11
(7,9)	14	6	59	9
(1,5)	30	8	53	7
(3,7)	103	15	85	13
(5,9)	8	5	45	9
(1,7)	41	11	11	5
(3,9)	46	11	25	6
(1,9)	35	10	121	11

Table 3: What Fraction of Writers did not Write Contracts

Periods	US b=1.25	US b=2.25	Predicted
1-5	0.222	0.172	0
6-10	0.267	0.269	0
11-15	0.311	0.290	0
16-20	0.326	0.303	0
21-25	0.356	0.283	0
26-30	0.333	0.283	0
Overall	0.302	0.267	0

Table 4: What Fraction of Writers did not Write Contracts in at Least 10/20 of the 30 Periods Played

Minimum Number of Periods	b=1.25 (N=27)	b=2.25 (N=29)
10	0.333	0.310
20	0.259	0.241

As can be seen in figure 2, when restricting the sample to only cases where a writer writes a contract, it includes the states 1 or 9 57.9% of the time in the b=1.25 treatment and 64.4% of the time in the b=2.25 treatment. Thus, this prediction is strongly rejected.

Result 3. Prediction three is not supported. Subjects are classified as having approximately correct actions given their contracts as long as the action in the contract is within ± 1 of the correct action given the contract. This classification only captures 51.24% of the data in the $b = 1.25$ treatment and 70.95% of the data in the $b = 2.25$ treatment. In addition, when using a rank-sum test to see whether the distributions of writer action errors are the same across both treatments, the hypothesis that the two groups of subjects have similar distributions of writer action errors relative to the contract written is rejected at the 1% level.

The total distribution of errors can be seen in figure 1. The average absolute error in the $b = 1.25$ treatment was 1.918, while the average absolute error in the $b = 2.25$ treatment was 1.046. A rank-sum test for a difference in means yielded a p-value of 5.268×10^{-63}, so I can reject the null hypothesis that players make the same errors in the two treatments.

This result that can be explained in one of two ways: Either writers have preferences that are biased towards their receiving partners, or this particular part of the task is heavily influenced by the sophistication of the subjects. Looking at the distribution, many of the errors are in the positive direction, indicating that the is not a result of preferences such as guilt or positive reciprocity, which negative errors might indicate.

When comparing the two treatments, the distributions of differences between actual action and predicted action are different at the 1% level using a rank-sum test. This indicates a difference between how subjects were thinking about the problem between the two treatments. Why this may be the case is an important question. It is possible that, once again, this has to do with sophistication, as this difference disappears in the Japan treatment.

Result 4. Prediction four is supported. Using a rank-sum test, for all data, the distribution of the number of states included in contract is different at the 1% level. In addition, periods 26-30 differ at the 5% level, while periods 21-25 differ at the 10% level.

The average number of states included in the contract starts off at a similar point and diverges after many periods of play. However, as is shown in the results of figure 5, using a rank-sum test for each group of 5 periods yields a significant difference for the last two periods in the sample. In addition, if all periods of both treatments are compared, there is a difference at the 1% level. This evidence is not the strongest, as the rank-sum test treats each individual period as a separate data point, ignoring correlation by individuals. In addition, there is no significance at the one-period level. However, because learning goes

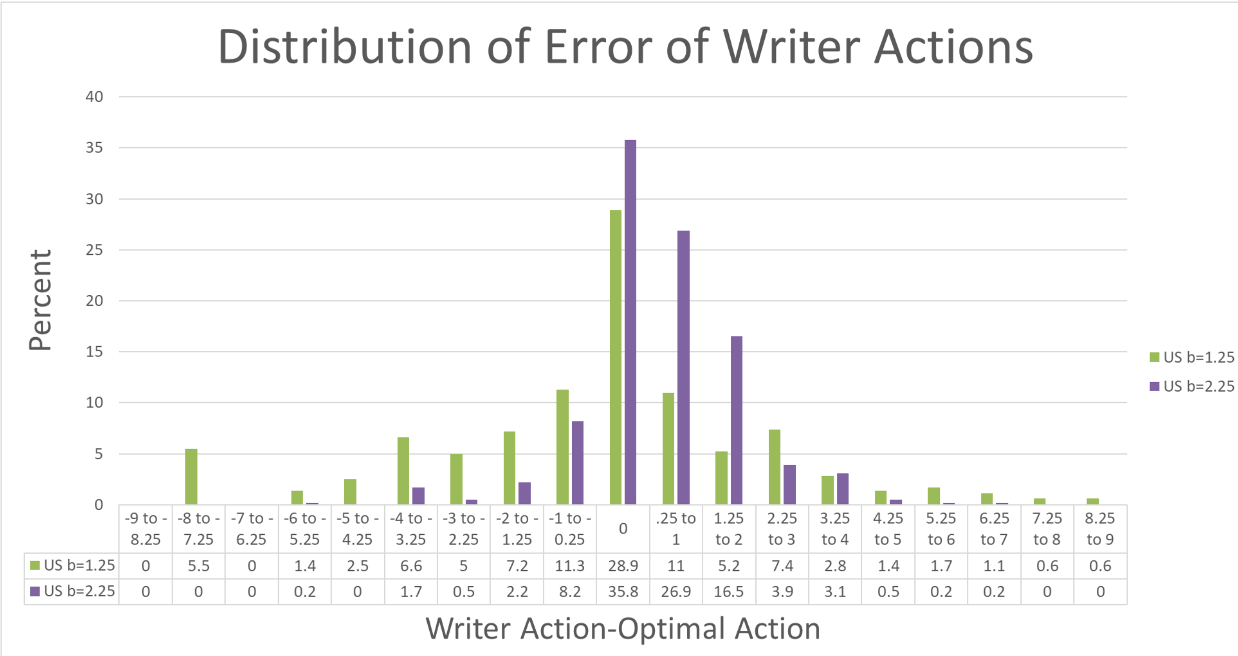


Figure 1: Difference Between the Optimal Writer Action and the Actual Writer Action Given the Contract (Excluding First 10 Periods)

Table 5: Average Number of States in Contract

	b=1.25		b=2.25		
Periods	N	Mean	N	Mean	p-value
1-5	135	2.356 (0.142)	145	2.497 (0.137)	0.23348
6-10	135	1.963 (0.132)	145	2.007 (0.144)	0.51443
11-15	135	1.815 (0.131)	145	2.021 (0.142)	0.18419
16-20	135	1.637 (0.122)	145	1.862 (0.140)	0.20387
21-25*	135	1.474 (0.119)	145	1.779 (0.132)	0.08651
26-30**	135	1.467 (0.119)	145	1.910 (0.141)	0.02714
All Periods***	810	1.785 (0.184)	870	2.013 (0.154)	0.00974

*, **, and *** indicate a significant difference between means at the 10%, 5%, and 1% levels using a Mann-Whitney U (rank-sum) test. Standard errors are in parenthesis.

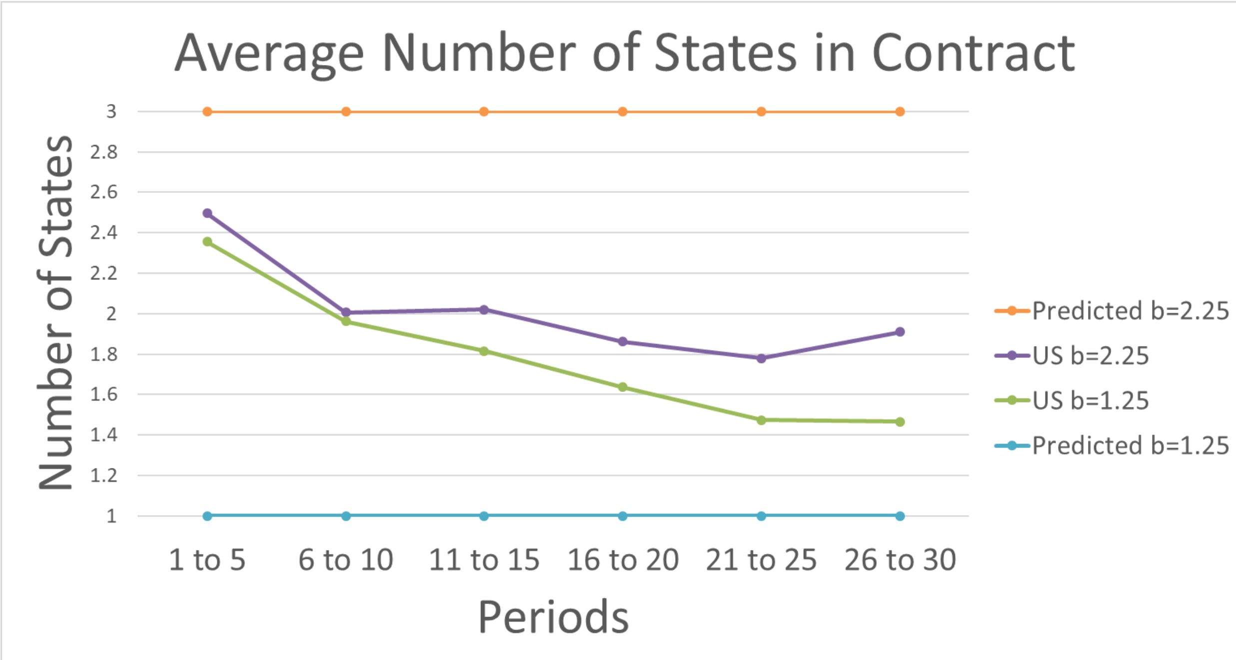


Figure 2: Average Number of States in Contract

in the correct direction, and because there is significant difference in the aggregate and in the last set of five periods, this constitutes evidence in support of prediction four.

Looking at the data, it is clear that the number of states included in the contract is far from what is predicted in the analysis on optimal contracts. Interestingly, the two treatments seem to have errors in opposite directions: In the low-bias treatment, more states are included on average than what is predicted. In the high-bias treatment, fewer states are included on average than what is predicted.

4.2 Behavior Within Communication Subgames and Impacts on Contract Writing

The goal of this section will be to provide some insight as to why writers do not write contracts like the theory predicted. The main result of this section is that, in many communication subgames, the actual average payoff is significantly higher than predicted. In addition, compounded with the error that comes with subjects writing actions in contracts, subjects who relied on more communication tended to do better in both treatments.

In looking at prediction five, I analyzed communication subgames which were reached by many subjects and had many data points. For many of the contracts, the sample size was too small to make any meaningful predictions, so I chose communication subgames that were reached regularly and by many different subjects. Unfortunately, only six unique subjects wrote a contract with $s_{low} = s_{high} = 3$, with four unique subjects writing the contract with $s_{low} = s_{high} = 7$, so analysis would not be meaningful for those communication subgames.

Result 5. Prediction five is not supported. Overall, play is not consistent with the most informative perfect Bayesian Nash equilibrium.

This result was obtained with the assumption that writers randomize uniformly over any messages that are numbers contained within a partition element within a communication subgame. When analyzing many communication subgames with sufficiently high data, the state-message, message-action, and state-action correlations are statistically different than the most informative communication equilibrium using a t-test for differences in correlation.[4] This is not true of all communication subgames analyzed in the paper. Notably, the communication subgame after $s_{low} = 5$, $s_{high} = 7$ for $b = 2.25$ has communication that is not significantly different than the most informative perfect Bayesian Nash equilibrium. However, the fact that this occurs for only one of the two treatments still supports the conclusion that overall play is significantly different from predicted.

When looking at table 6, note that it is not true that writers always overcommunicate and that receivers overly believe the writer. In many communication subgames, there is more muddled communication than is predicted by the model, or else the receiver believes the writer less than what is predicted. This is particularly harmful in communication subgames like the one that occurs after the contract $C = \{3, 7, a_W\}$, where there should be completely honest communication. However, what happens is far from honest, as there is only a .448 correlation between state and action in the $b = 1.25$ treatment and a .690 correlation in the $b = 2.25$ treatment, both of which are statistically lower than what is predicted using a t-test. This is contrary to a well-known result in Cai & Wang (2006), where the authors find that overcommunication is common.

[4] Following Cai & Wang, the regression $Y = \alpha + (r_{XY} + \beta)\, X + \epsilon$ was run, where $r_{XY} = (s_Y/s_X)\, \sigma_{XY}$, s_X and s_Y are the sample standard deviations of X and Y and σ_{XY} is the theoretical correlation. The t-test on β would say whether the actual correlation $Corr(X, Y)$ is statistically different from the theoretical correlation σ_{XY}. The regressions were run with (Y, X) being one of (State, Message), (Message, Action), and (State, Action). This was done for many different communication subgames, such that there were many different subjects and data points. Some with fewer subjects/data points were also included to try to cover more communication subgames.

Table 6: Correlations in Selected Communication Subgames

Communication Subgame Following the Contract (Low State, High State)	b	# of Times Communication Subgame is Reached.	Predicted State-Message	Actual State-Message	Predicted Message-Action	Actual Message-Action	Predicted State-Action	Actual State-Action
No Contract	1.25	245	0.776	0.848**	0.896	0.792***	0.866	0.674***
No Contract	2.25	232	0.527	0.643**	0.745	0.751	0.707	0.555***
(5,5)	2.25	29	0.915	0.894	0.965	0.802	0.949	0.694*
(1,3)	1.25	55	0.783	0.495**	0.905	0.401***	0.866	0.416***
(1,3)	2.25	25	0	0.605***	0	0.598***	0	0.299*
(3,5)	1.25	55	0.951	0.560***	0.980	0.611***	0.971	0.315***
(5,7)	1.25	15	0.952	0.381**	0.980	0.531**	0.971	0.368**
(5,7)	2.25	21	0.952	0.944	0.980	0.830	0.971	0.748
(3,7)	1.25	56	1	0.624***	1	0.755***	1	0.448***
(3,7)	2.25	36	1	0.791**	1	0.821**	1	0.690***

*, **, and *** indicate a significant difference from the predicted values at the 10%, 5%, and 1% levels using a t-test for differences in correlation

Table 7: Writer Payoff Given Contract

(Low State, High State)	b=1.25			b=2.25		
	n	Average Payoff	Predicted Payoff	n	Average Payoff	Predicted Payoff
No Contract	245	27.4	26.438	232	26.066	20.938
(1,1)	25	26.063	27.15	12	25.483	21.938
(3,3)	19	23.975	28.35	31	24.907	24.35
(5,5)	21	28.103	27.95	37	25.800	25.15
(7,7)	5	29.19	28.35	15	26.732	24.15
(9,9)	34	24.752	27.15	48	28.139	21.95
(1,3)	94	26.928	28.265	37	25.015	24.962
(3,5)	72	24.862	28.265	20	25.110	26.162
(5,7)	18	24.363	28.265	39	27.173	26.162
(7,9)	14	27.653	28.265	59	28.231	24.962
(1,5)	30	27.146	27.375	53	25.927	25.975
(3,7)	103	25.076	27.775	85	26.755	26.375
(5,9)	8	26.781	27.375	45	26.438	25.975
(1,7)	41	23.703	25.6875	11	22.777	24.988
(3,9)	46	24.373	25.6875	25	24.532	24.988
(1,9)	35	24.716	22	121	25.450	22

Result 6. Prediction six is supported. Subjects tended to switch contracts more often when they earned a poor payoff in the previous round.

The behavior inside communication subgames explains some of the behavior of writers in the previous subsection. Communication games where very incomplete contracts are written did better than communication in many other communication subgames, as can be observed in table 7. This means that writers were strongly incentivized to abstain from writing a contract. A key question is whether writers wrote contracts taking this into account or whether communication was influenced by the writing of the contract. Although the latter does not seem true, the former seems to have some merit due to the large number of people writing no contract. In table 8, it is evident that more subjects switch the contract they write after receiving a bad payoff, indicating that subjects are responsive to receiving bad payoffs.

Table 8: Did Writers Switch Contracts when Receiving Bad Payoffs

Payoff Received in Previous Period	% Stayed with the Same Contract in the Following Period	
	$b = 1.25$	$b = 2.25$
> 27 ECUs	69.2% (0.031)	71.4% (0.024)
≤ 27 ECUs	43.1% (0.020)	64.3% (0.022)
t Statistic	7.273***	2.190**

*, **, and *** indicate a significant difference from the predicted values at the 10%, 5%, and 1% levels using a two sample t-test assuming equal variances. Standard errors are in parenthesis.

4.3 Robustness Check: Experimental Results of Sessions in Japan

In addition to the primary treatments, two sessions were run at Waseda University. This serves as a good check on whether the sophistication of subjects impacts the results of the experiment.

Because the sample size is too small, results regarding behavior in communication subgames are mainly useless, as each communication subgame is only reached by five or fewer total subjects. Thus, this subsection will focus primarily on how predictions 1-4 are impacted.

Result 7. The sophistication of subjects impacts how subjects play in the following way: Writer actions contain far less error and are similar in error across treatments. Fewer subjects chose to not write a contract in the b=2.25 treatment.

For prediction one, as can be seen in table 9, in the b=2.25 treatment, far fewer subjects chose not to write a contract, although the number is still statistically significantly different from the predicted amount, zero. For prediction three, distributions of action errors are smaller (average absolute errors of 0.715 in the $b = 1.25$ treatment and 0.656 in the $b = 2.25$ treatment) and are statistically different at the 10% level when using a rank-sum test (p-value of 0.08971). In the Japanese treatment, the errors tend to skew downwards in the direction of an inequality-averse writer who would be more likely to include actions that favor the receiver. In addition, as can be observed in figure 3, the errors have a much tighter distribution. For prediction four, in each group of five periods, the number of states included in a contract is significantly different at some level, which can be seen in table 11.

The key factor that likely influences the stark difference between how contracts are written in the two subject groups is subject sophistication. According to the director of the ESL at Waseda University, Yukihiko Funaki, the average CRT (Cognitive Reflection Test) score of the Waseda subject pool is 2.02. Charles Noussair, the director of the ESL at the Univer-

Table 9: What Fraction of Writers Did Not Write Contracts in Japan Sessions

Periods	US b=1.25	US b=2.25	JPN b=1.25	JPN b=2.25	Predicted
1-5	0.222	0.172	0.2	0.12	0
6-10	0.267	0.269	0.309	0.12	0
11-15	0.311	0.290	0.291	0.02	0
16-20	0.326	0.303	0.327	0.06	0
21-25	0.356	0.283	0.291	0.1	0
26-30	0.333	0.283	0.273	0.04	0
Overall	0.302	0.267	0.282	0.077	0

sity of Arizona, states that the average CRT score of the University of Arizona subjects is around 0.8. Calculating the correct action to use is mathematical in nature, and so a logical conclusion is that subjects with a higher CRT would do better at writing correct actions. In addition, there seemed to be a more powerful learning effect in the University of Arizona treatments than in the Waseda treatments, indicating that in the Japanese treatments, subjects more quickly grasped ideas about how to write contracts.

State-message, message-action, and state-action correlations are also analyzed. These are found in table 12. For the small sample size that is available, communication after no contract was written is closer to the results of Cai & Wang, while communication with one state on each side is even worse than before, with a state-action correlation of 0.161 in the $b = 1.25$ treatment and a state-action correlation of 0.251 in the $b = 2.25$ treatment. This provides additional evidence that undercommunication may occur in communication subgames, and that undercommunication may influence behavior.

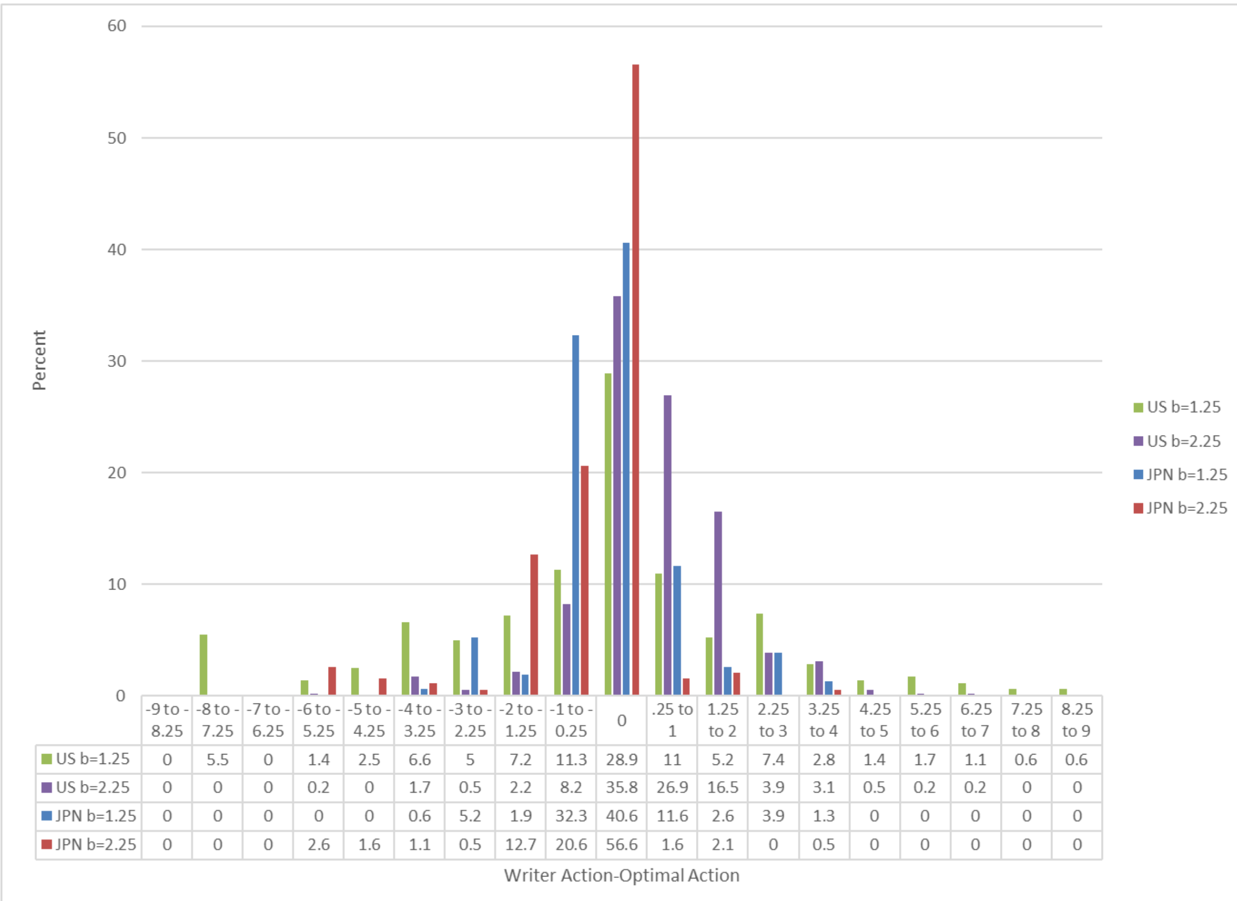
Table 10: Events that Writers Wrote in Contracts in Japan Sessions

(Low State, High State)	b=1.25		b=2.25	
	Total Instances	Number of Unique Subjects	Total Instances	Number of Unique Subjects
No Contract	93	5	23	5
(1,1)	6	2	4	1
(3,3)	24	3	4	4
(5,5)	2	2	12	3
(7,7)	0	0	0	0
(9,9)	7	4	59	3
(1,3)	25	5	1	1
(3,5)	13	3	12	4
(5,7)	46	5	39	4
(7,9)	49	5	12	5
(1,5)	12	4	11	2
(3,7)	21	2	52	4
(5,9)	9	3	50	4
(1,7)	7	2	1	1
(3,9)	10	3	14	3
(1,9)	6	3	6	4

Table 11: Average Number of States in Contract in Japanese Sessions

	b=1.25		b=2.25		
Periods	N	Mean	N	Mean	p-value
1-5*	55	1.927 (0.189)	50	2.34 (0.199)	0.05158
6-10***	55	1.509 (0.162)	50	2.16 (0.177)	0.00430
11-15***	55	1.582 (0.131)	50	2.26 (0.139)	0.00262
16-20**	55	1.491 (0.170)	50	2 (0.148)	0.01072
21-25*	55	1.618 (0.173)	50	1.96 (0.162)	0.06873
26-30***	55	1.491 (0.168)	50	2 (0.140)	0.00842
All Periods***	330	1.603 (0.184)	300	2.12 (0.280)	4.76×10^{-8}

*, **, and *** indicate a significant difference between means at the 10%, 5%, and 1% levels using a Mann-Whitney U (rank-sum) test. Standard errors are in parenthesis.



	-9 to -8.25	-8 to -7.25	-7 to -6.25	-6 to -5.25	-5 to -4.25	-4 to -3.25	-3 to -2.25	-2 to -1.25	-1 to -0.25	0	.25 to 1	1.25 to 2	2.25 to 3	3.25 to 4	4.25 to 5	5.25 to 6	6.25 to 7	7.25 to 8	8.25 to 9
US b=1.25	0	5.5	0	1.4	2.5	6.6	5	7.2	11.3	28.9	11	5.2	7.4	2.8	1.4	1.7	1.1	0.6	0.6
US b=2.25	0	0	0	0.2	0	1.7	0.5	2.2	8.2	35.8	26.9	16.5	3.9	3.1	0.5	0.2	0.2	0	0
JPN b=1.25	0	0	0	0	0	0.6	5.2	1.9	32.3	40.6	11.6	2.6	3.9	1.3	0	0	0	0	0
JPN b=2.25	0	0	0	2.6	1.6	1.1	0.5	12.7	20.6	56.6	1.6	2.1	0	0.5	0	0	0	0	0

Figure 3: Difference Between the Optimal Writer Action and the Actual Writer Action Given the Contract in Japan and US Treatments

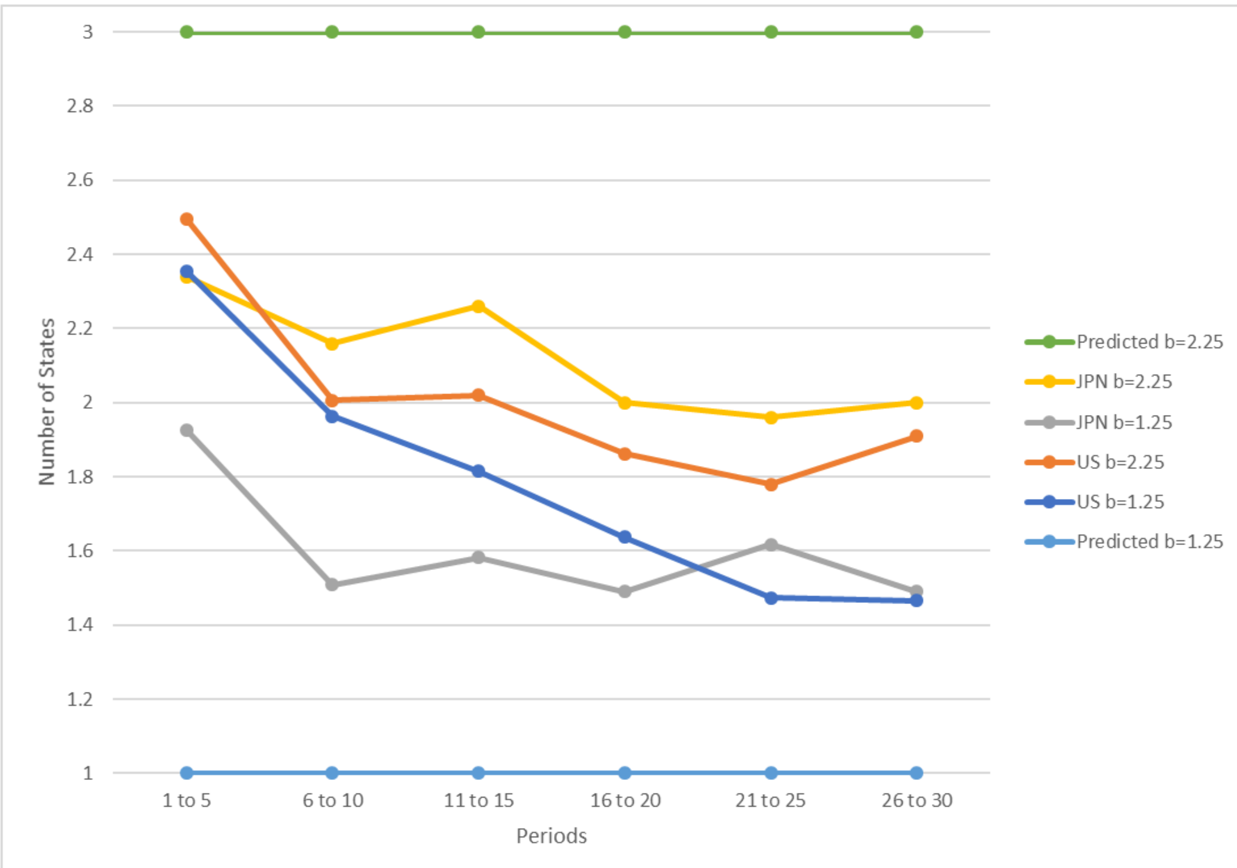


Figure 4: Number of States Included in Contracts in Japan and US Sessions

Table 12: Correlations in Each Communication Subgame

Communication Subgame Following the Contract (Low State, High State)	b	# of Times Communication Subgame is Reached.	Predicted State-Message	Actual State-Message	Predicted Message-Action	Actual Message-Action	Predicted State-Action	Actual State-Action
No Contract	1.25	93	0.776	0.919***	0.896	0.945	0.866	0.857
No Contract	2.25	23	0.527	0.572	0.745	0.629	0.707	0.641
(3,7)	1.25	21	1	0.198**	1	0.881	1	0.161**
(3,7)	2.25	52	1	0.554**	1	0.767	1	0.251***

*, **, and *** indicate a significant difference from the predicted values at the 10%, 5%, and 1% levels using a t-test for differences in correlation

5 Conclusion

This paper is an initial glimpse into explaining how people write incomplete contracts when incompleteness is predicted to be optimal. The number of states included in the contract increases as bias increases, validating theoretical predictions in this paper and reinforcing messages in related papers Shavell (2006) and Heller & Spiegler (2008). Subjects wrote contracts that did not include the correct writer action given the states in the contract. Subjects in the Japanese treatment wrote contracts that were more closely aligned with predictions. It is hypothesized that this has to do with subject pool sophistication.

Additionally, this paper analyzes the relationship between the interpretation process and the writing of incomplete contracts. Some writers tended to abstain from writing contracts, and payoffs in that communication subgame were better than predicted. In general, communication subgames in which the contract specified states strictly in the middle of the state space yielded poor payoffs to subjects. Furthermore, there was a general theme of undercommunication in the communication subgames analyzed in this paper.

There are many avenues for future research that build off of this experiment. Although this paper explores how people go about writing contracts and how people go through the interpretation process presented, there is an unclear causal link between the two. It is unclear whether behavior inside of communication subgames causes different behavior in contract writing or whether behavior in contract writing drives the way players communicate. It would be revealing to analyze experiments that fix either the contract or the interpretation process to isolate how people play in absence of one of the aspects of the experiment. These kinds of explorations may also help explain why undercommunication is common in communication subgames and why communication results appear to differ from Cai & Wang (2006).

This task is difficult in similar ways to contract writing. Not only do subjects have to do calculations to determine the best action, they must also keep in mind the tradeoff between including states in a contract and not including states in a contract. This paper is one of the first attempts at exploring how subjects behave in an experiment where another party providing interpretation for gaps in the contract provides an incentive to write an incomplete contract. Hopefully, this project inspires other researchers to explore how people write incomplete contracts.

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Appendix A: Misc Tables and Graphs

In this section I will include tables and graphs that are not directly relevant to the main text that may be of interest.

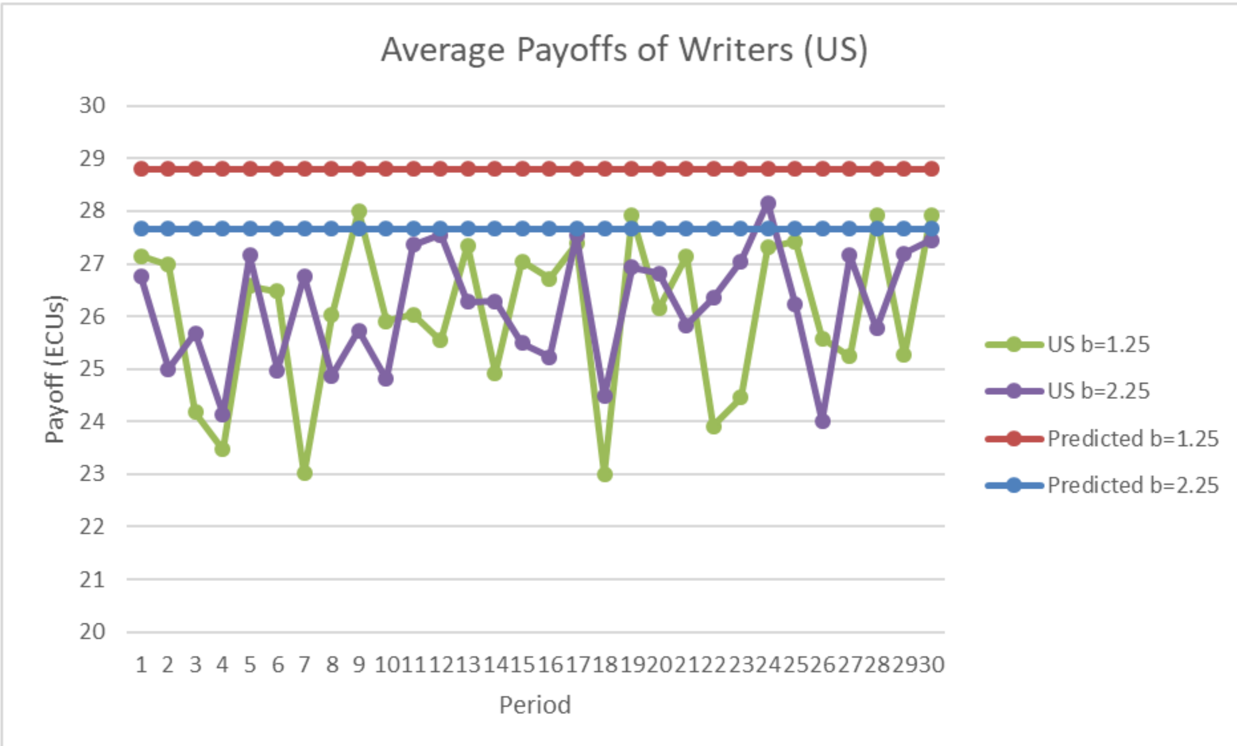


Figure 5: Average Writer Payoff of US Subjects

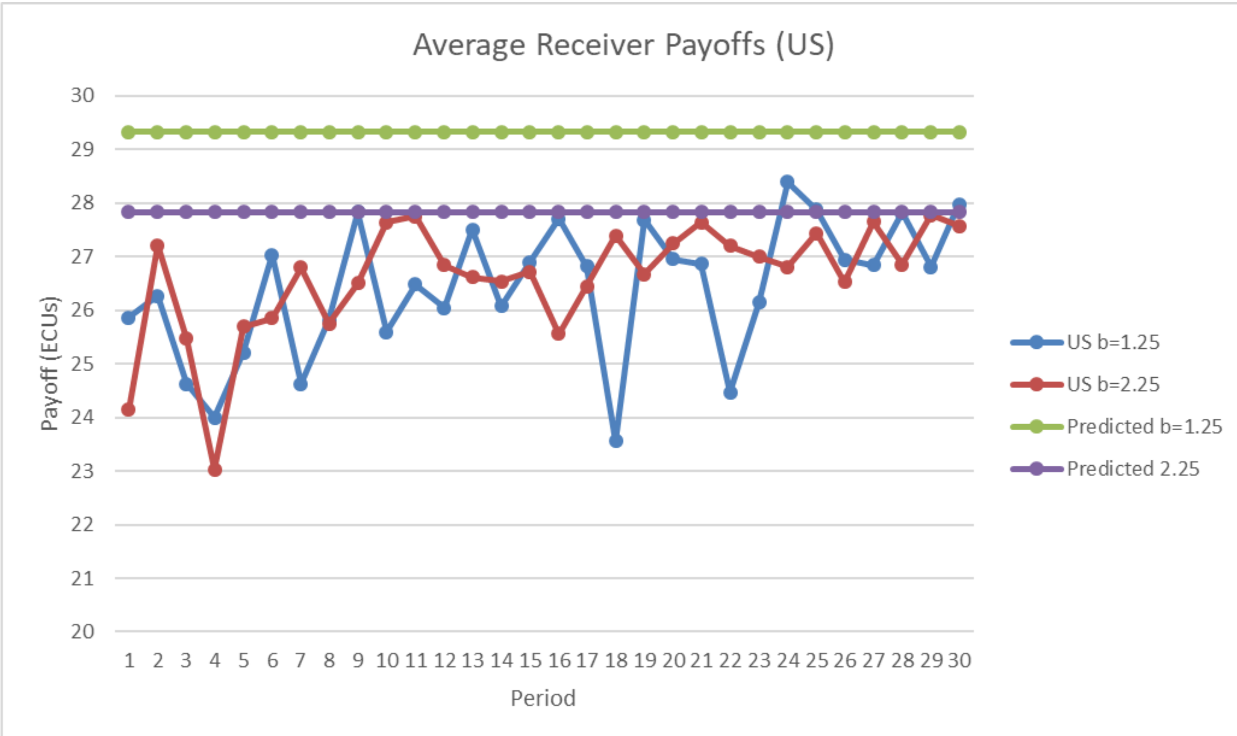


Figure 6: Average Receiver Payoff of US Subjects

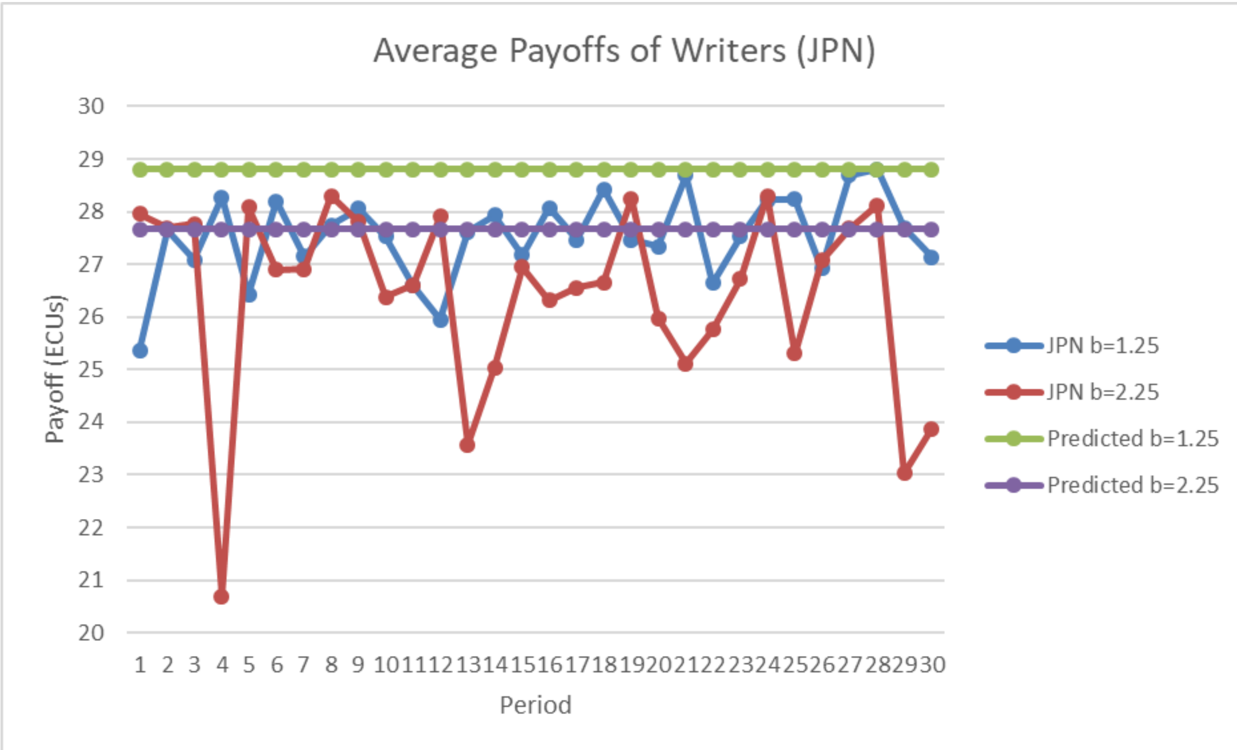


Figure 7: Average Writer Payoff of JPN Subjects

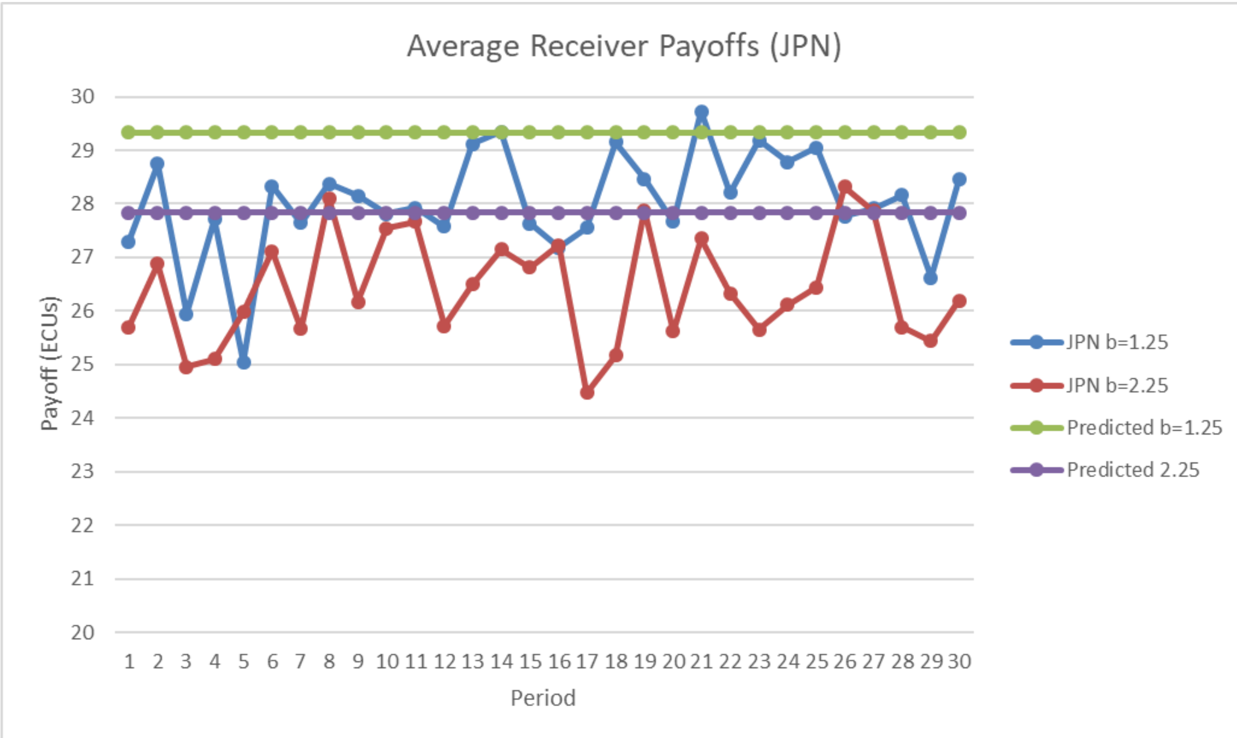


Figure 8: Average Receiver Payoff of JPN Subjects

Table 13: Writer Payoff Given Communication Subgame is Reached

	b=1.25		b=2.25	
(Low State, High State)	Average Payoff	Predicted Payoff	Average Payoff	Predicted Payoff
No Contract	27.4	26.438	26.066	20.938
(1,1)	25.526	26.438	25.072	19.938
(3,3)	24.3	27.938	24.250	22.938
(5,5)	27.573	27.438	25.399	23.938
(7,7)	29.19	27.938	26.109	22.938
(9,9)	27.476	26.438	27.442	19.938
(1,3)	26.178	27.771	23.423	22.271
(3,5)	24.269	27.771	24.964	24.271
(5,7)	23.978	27.771	26.918	24.271
(7,9)	28.184	27.771	27.871	22.271
(1,5)	26.635	27.438	23.598	23.938
(3,7)	24.206	28.438	25.444	24.938
(5,9)	25.87	27.438	26.621	23.938
(1,7)	22.47	28.438	20.674	24.938
(3,9)	26.376	28.438	27.384	24.938
(1,9)	N/A	N/A	N/A	N/A

Table 14: Receiver Payoff Given Communication Subgame is Reached

	b=1.25		b=2.25	
(Low State, High State)	Average Payoff	Predicted Payoff	Average Payoff	Predicted Payoff
No Contract	27.847	28	26.790	26
(1,1)	27.089	28	27.537	25
(3,3)	25.958	29.5	27.213	28
(5,5)	27.222	29	27.396	29
(7,7)	28.986	29.5	27.74	28
(9,9)	28.068	28	27.198	25
(1,3)	27.451	29.333	26.819	27.333
(3,5)	25.066	29.333	27.932	29.333
(5,7)	25.052	29.333	27.129	29.333
(7,9)	26.914	29.333	27.427	27.333
(1,5)	26.718	29	27.319	29
(3,7)	24.797	30	26.629	30
(5,9)	24.3	29	26.18	29
(1,7)	24.573	30	25.6	30
(3,9)	24.973	30	27.696	30
(1,9)	N/A	N/A	N/A	N/A

Appendix B: Proofs for Observations

Each observation will be stated below for convenience:

Observation 1: *Given a low state s_{low} and a high state s_{high}, the optimal contract specifies the writer action $a_w = \frac{s_{low}+s_{high}}{2} + b$. Conditional on the receiver knowing that the state $s \in S' \subseteq S$, the optimal receiver action is $\mathbb{E}[s \mid s \in S']$.*

Proof. The first statement is proved by writing down the first order condition for the expected utility problem and noting that the expected utility is concave. For a contract (s_{low}, s_{high}, a^*)

$$EU_W(\cdot) = \sum_{0}^{\frac{s_{high}-s_{low}}{2}} -\frac{1}{\frac{s_{high}-s_{low}}{2}+1}(s_{low}+2i+b-a^*)^2.$$

Taking the first derivative and setting it equal to 0 yields the equation

$$\sum_{0}^{\frac{s_{high}-s_{low}}{2}} \frac{2}{\frac{s_{high}-s_{low}}{2}}(s_{low}+2i+b-a^*) = 0.$$

Solving for a^* reduces the equation to

$$a^* = s_{low} + b + \frac{\sum_{1}^{\frac{s_{high}-s_{low}}{2}} 2i}{\frac{s_{high}-s_{low}}{2}+1} = s_{low} + b + \frac{\frac{2\left(\frac{s_{high}-s_{low}}{2}\right)\left(\frac{s_{high}-s_{low}}{2}+1\right)}{2}}{\frac{s_{high}-s_{low}}{2}+1}$$

$$= s_{low} + b + \frac{s_{high}-s_{low}}{2} = \frac{s_{high}+s_{low}}{2} + b.$$

The second statement is trivially true because the receiver is strictly risk averse and the payoff function is symmetric. ■

Observation 2: *In any communication subgame, a perfect Bayesian equilibrium of that subgame is represented as a partition $P = \{p_1, \ldots, p_n\}$ for $n \geq 1$, where $p_i = \left\{s_1^i, ..., s_{k_i}^i\right\}$ is a partition element such that a message $m \in M_i \subseteq M$ sent when $s^i \in p_i$ induces a unique expected action $a_R^i \neq a_R^j \forall j$, where a_R^i is the expected receiver preferred action given $m \in M_i$ is sent. Each partition element is ordered such that for any $s^i \in p_i$, $s^j \in p^j$, $s^i < s^j$.*

Proof. In any perfect Bayesian equilibrium of any communication subgame, for some subset of the message space $\hat{M}$ a writer must weakly prefer sending a message $m \in \hat{M}$ in state s to sending any other message $m' \in \left\{M \setminus \hat{M}\right\}$, given that a receiver best responds by playing the action $a_R = \mathbb{E}[s \mid m]$. Denote a set of states that induce the same expected action p_i. These p_i form the partition $P = \{p_1, \ldots, p_n\}$. Denote the set of messages that can possibly be sent if $s \in p_i$ by M_i. Because the actions in any two partition elements differ and because the single crossing condition holds, for $a_j > a_i$ if a writer prefers action a_i in state $s \in p_i$ and a writer prefers action a_j in state $s' \in p_j$, then for any $s'' > s'$ a writer must prefer a_j to a_i, meaning that p_i cannot contain any states larger than any state in p_j. ■

Observation 3: *A writer will always choose to write a contract.*

Proof. Suppose that the writer does not write a contract. Suppose that there is some selected equilibrium after no contract is written that has a partition $P = \{p_1, \ldots, p_n\}$ for $n \geq 1$. Let the partition elements be ordered based on the lowest state in each partition element, such that $s_1^1 < s_1^2 < \ldots < s_1^n$. Suppose that the writer had written the contract $C = \left(s_{low}^1, s_{high}^1, a_W^1\right)$ where a_W^1 is the writer preferred action given $s \in p_1$. Note that if there is a resulting communication subgame, $P^C = \{p_2, \ldots, p_n\}$ is a perfect Bayesian equilibrium of the communication subgame because all messages sent by states inside p_1 were unique, so there are still no incentives for the writer to change messages given that incentive constraints between the remaining partition elements have not changed. Therefore, there exists an equilibrium of the resulting communication subgame after C that makes the writer strictly better off since $a_W^i - a_R^i > 0$ for $b \geq 1$. Thus writing a contract must be optimal. ■

Appendix C: Calculating Optimal Contracts

In order to write down the optimal contract for any bias, I need to analyze every communication equilibrium in every possible communication subgame. Firstly, because of proposition 3, I can ignore the communication subgame after no contract is written since not writing a contract is never optimal for any b. Secondly, I can use proposition 1 to pin down $a_W = \frac{s_{low}+s_{high}}{2}$. In addition, given any partition P, I can use proposition 1 to pin down the receiver action that happens in that partition element. Using each of these properties, it remains to write down and compare the payoffs for any possible communication subgame to figure out what contract the writer will choose in period 1. Below, I will list all possible $n-state$ contracts, for $n \geq 1$, as well as the accompanying picture that shows which contract and communication subgame pair does better. After exhausting all possible communication subgames, I will compare the winners in each $n-state$ contract to get the optimal contract. As a further note, since $EU\left(C = \{1, 9, 5+b\}, P = \emptyset\right) = 26.16$, once payoffs dip below 26.16 the fully complete contract does better.

As another note, contracts that are symmetric around 5 can have equivalent sets of communication equilibria within communication subgames and thus equivalent payoffs. Thus only one of the two will have equilibria and payoffs presented.

$1 - \textit{state}$ contracts:

$C = \{1, 1, 1+b\}$(symmetric to $C = \{9, 9, 9+b\}$):

$(b \le 1)$ $P = \{\{3\}, \{5\}, \{7\}, \{9\}\}$: $EU_W(C, P) = 30 - \frac{4}{5}|b|^{1.4}$

$(b \le 1)$ $P = \{\{3, 5\}, \{7, 9\}\}$: $EU_W(C, P) = 30 - \frac{2}{5}|b-1|^{1.4} - \frac{2}{5}|b+1|^{1.4}$

$(b \le 2)$ $P = \{\{3\}, \{5, 7, 9\}\}$: $EU_W(C, P) = 30 - \frac{1}{5}|b-2|^{1.4} - \frac{2}{5}|b|^{1.4} - \frac{1}{5}|b+2|^{1.4}$

(all b) $P = \{\{3, 5, 7, 9\}\}$: $EU_W(C, P) = 30 - \frac{1}{5}|b-3|^{1.4} - \frac{1}{5}|b-1|^{1.4} - \frac{1}{5}|b+1|^{1.4} - \frac{1}{5}|b+1|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{3\}, \{5\}, \{7\}, \{9\}\} & b \le 1 \\ P = \{\{3\}, \{5, 7, 9\}\} & 1 < b \le 2 \\ P = \{\{3, 5, 7, 9\}\} & b > 2 \end{cases}$$

$C = \{3, 3, 3+b\}$(symmetric to $C = \{7, 7, 7+b\}$):

$(b \le 1)$ $P = \{\{1\}, \{5\}, \{7\}, \{9\}\}$: $EU_W(C, P) = 30 - \frac{4}{5}|b|^{1.4}$

$(b \le 1.5)$ $P = \{\{1\}, \{5\}, \{7, 9\}\}$: $EU_W(C, P) = 30 - \frac{1}{5}|b-1|^{1.4} - \frac{2}{5}|b|^{1.4} - \frac{1}{5}|b+1|^{1.4}$

$(b \le 3)$ $P = \{\{1\}, \{5, 7, 9\}\}$: $EU_W(C, P) = 30 - \frac{1}{5}|b-2|^{1.4} - \frac{2}{5}|b|^{1.4} - \frac{1}{5}|b+2|^{1.4}$

(all b) $P = \{\{1, 5, 7, 9\}\}$: $EU_W(C, P) = 30 - \frac{1}{5}|b - \frac{9}{2}|^{1.4} - \frac{1}{5}|b - \frac{1}{2}|^{1.4} - \frac{1}{5}|b + \frac{3}{2}|^{1.4} - \frac{1}{5}|b + \frac{7}{2}|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{1\}, \{5\}, \{7\}, \{9\}\} & b \le 1 \\ P = \{\{1\}, \{5\}, \{7, 9\}\} & 1 < b \le 1.5 \\ P = \{\{1\}, \{5, 7, 9\}\} & 1.5 < b \le 3 \\ P = \{\{1, 5, 7, 9\}\} & b > 3 \end{cases}$$

$C = \{5, 5, 5+b\}$:

$(b \le 1)$ $P = \{\{1\}, \{3\}, \{7\}, \{9\}\}$: $EU_W(C, P) = 30 - \frac{4}{5}|b|^{1.4}$

$(b \le 2)$ $P = \{\{1, 3\}, \{7, 9\}\}$: $EU_W(C, P) = 30 - \frac{2}{5}|b-1|^{1.4} - \frac{2}{5}|b+1|^{1.4}$

($b \leq \frac{11}{3}$) $P = \{\{1\}, \{3,7,9\}\}$: $EU_W(C,P) = 30 - \frac{1}{5}|b - \frac{10}{3}|^{1.4} - \frac{1}{5}|b|^{1.4} - \frac{1}{5}|b + \frac{2}{3}|^{1.4} - \frac{1}{5}|b + \frac{8}{3}|^{1.4}$

(all b) $P = \{\{1,3,7,9\}\}$: $EU_W(C,P) = 30 - \frac{1}{5}|b-4|^{1.4} - \frac{1}{5}|b-2|^{1.4} - \frac{1}{5}|b + 2|^{1.4} - \frac{1}{5}|b+4|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{1\}, \{3\}, \{7\}, \{9\}\} & b \leq 1 \\ P = \{\{1,3\}, \{7,9\}\} & 1 < b \leq 2 \\ P = \{\{1\}, \{3,7,9\}\} & 2 < b \leq \frac{11}{3} \\ P = \{\{1,3,7,9\}\} & b > 3 \end{cases}$$

Overall best $1 - state$ payouts:

$$\begin{cases} \text{Any contract with the most informative equilibrium} & b \leq 1 \\ C = \{3,3,3+b\}, P = \{\{1\}, \{5\}, \{7,9\}\} & 1 < b \leq 1.5 \\ C = \{5,5,5+b\}, P = \{\{1,3\}, \{7,9\}\} & 1.5 < b \leq 2 \\ C = \{3,3,3+b\}, P = \{\{1\}, \{5,7,9\}\} & 2 < b \leq 3 \\ C = \{1,1,1+b\}, P = \{\{3,5,7,9\}\} & b > 3 \end{cases}$$

$2 - state$ contracts:

$C = \{1,3,2+b\}$(symmetric to $C = \{7,9,8+b\}$):

($b \leq 1$) $P = \{\{5\}, \{7\}, \{9\}\}$: $EU_W(C,P) = 29.6 - \frac{3}{5}|b|^{1.4}$

($b \leq 1.5$) $P = \{\{5\}, \{7,9\}\}$: $EU_W(C,P) = 29.6 - \frac{1}{5}|b-1|^{1.4} - \frac{1}{5}|b|^{1.4} - \frac{1}{5}|b+1|^{1.4}$

(all b) $P = \{\{5,7,9\}\}$: $EU_W(C,P) = 29.6 - \frac{1}{5}|b-2|^{1.4} - \frac{1}{5}|b|^{1.4} - \frac{1}{5}|b+2|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{5\}, \{7\}, \{9\}\} & b \leq 1 \\ P = \{\{5\}, \{7,9\}\} & 1 < b \leq 1.5 \\ P = \{\{5,7,9\}\} & b > 1.5 \end{cases}$$

$C = \{3,5,4+b\}$(symmetric to $C = \{5,7,6+b\}$):

($b \leq 1$) $P = \{\{1\}, \{7\}, \{9\}\}$: $EU_W(C,P) = 29.6 - \frac{3}{5}|b|^{1.4}$

($b \leq 3.5$) $P = \{\{1\}, \{7,9\}\}$: $EU_W(C,P) = 29.6 - \frac{1}{5}|b-1|^{1.4} - \frac{1}{5}|b|^{1.4} - \frac{1}{5}|b+1|^{1.4}$

(all b) $P = \{\{1,7,9\}\}$: $EU_W(C,P) = 29.6 - \frac{1}{5}|b - \frac{14}{3}|^{1.4} - \frac{1}{5}|b + \frac{4}{3}|^{1.4} - \frac{1}{5}|b + \frac{10}{3}|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{1\},\{7\},\{9\}\} & b \le 1 \\ P = \{\{1\},\{7,9\}\} & 1 < b \le 3.5 \\ P = \{\{1,7,9\}\} & b > 3.5 \end{cases}$$

Overall best $2 - state$ payouts:

$$\begin{cases} \text{Any contract with the most informative equilibrium} & b \le 1 \\ C = \{1,3,2+b\}, P = \{\{5\},\{7,9\}\} & 1 < b \le 1.5 \\ C = \{3,5,4+b\}, P = \{\{1\},\{7,9\}\} & 1 < b \le 3.5 \\ C = \{1,3,2+b\}, P = \{\{5,7,9\}\} & b > 3.5 \end{cases}$$

$3 - state$ payouts:

$C = \{1,5,3+b\}$(symmetric to $C = \{5,9,7+b\}$):

$(b \le 1)$ $P = \{\{7\},\{9\}\}$: $EU_W(C,P) = 28.944 - \frac{2}{5}|b|^{1.4}$

$(b \ge 1)$ $P = \{\{7,9\}\}$: $EU_W(C,P) = 28.944 - \frac{1}{5}|b-1|^{1.4} - \frac{1}{5}|b+1|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{7\},\{9\}\} & b \le 1 \\ P = \{\{7,9\}\} & b > 1 \end{cases}$$

$C = \{3,7,5+b\}$:

$(b \le 4)$ $P = \{\{1\},\{9\}\}$: $EU_W(C,P) = 28.944 - \frac{2}{5}|b|^{1.4}$

$(b \ge 4)$ $P = \{\{1,9\}\}$: $EU_W(C,P) = 28.944 - \frac{1}{5}|b-4|^{1.4} - \frac{1}{5}|b+4|^{1.4}$

Best equilibria:

$$\begin{cases} P = \{\{1\},\{9\}\} & b \le 4 \\ P = \{\{1,9\}\} & b > 4 \end{cases}$$

Overall best $3 - state$ payouts:

$$\begin{cases} C = \{1,5,3+b\}, P = \{\{7\},\{9\}\} & b \le 1 \\ C = \{3,7,5+b\}, P = \{\{1\},\{9\}\} & b \le 4 \\ C = \{1,5,3+b\}, P = \{\{7,9\}\} & b > 4 \end{cases}$$

$4 - state$ contracts:

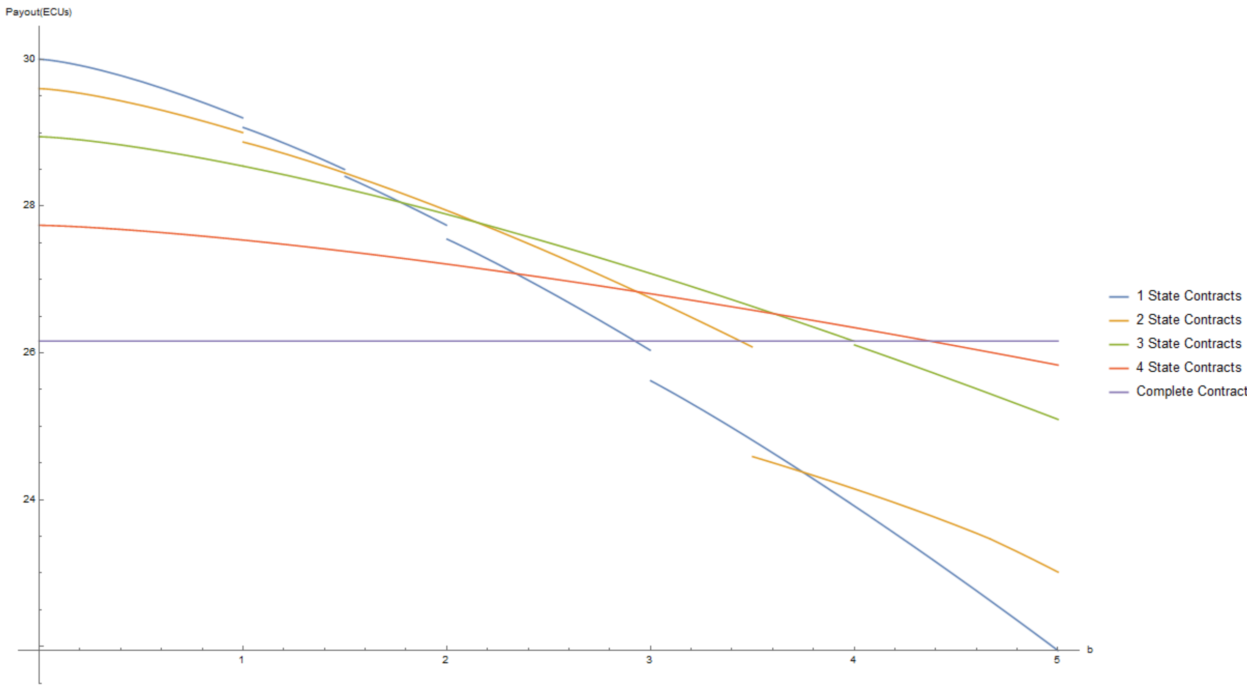


Figure 9: Comparison between all best $n - state$ contract/equilibrium partition pairs

$C = \{1, 7, 4 + b\}$(symmetric to $C = \{3, 9, 6 + b\}$):

(all b) $P = \emptyset$: $EU_W(C, P) = 27.738 - |b|^{1.4}$

$5 - state$ contract:

(all b) $P = \emptyset$: $EU_W(C, P) = 26.159$

Overall, the best contracts are detailed in table 1 when comparing among the best among each $n - state$ contract. This is shown graphically here in figure 9. Each line represents a piecewise function as described above.

Appendix D: Instructions

Below are the instructions for the US $b = 1.25$ treatment:

Welcome! In this experiment, your earnings will depend on your choices, the choices of others, and chance. Please refrain from talking to others until the experiment has concluded. In addition, please silence and put away any electronic devices (although listening to music is allowed).

The participants of this experiment will be randomly split between Writers and Receivers, such that half will be Writers and the other half will be Receivers. You will play only as a Writer or as a Receiver for the duration of the experiment. In each round, each Writer will be randomly paired with a Receiver. You will not see the identity of the person you are paired with, but you will see each player's decisions at the end of each round. Your total payment at the end of the experiment will be the sum of your earnings across the 9 paid rounds of the game.

Your payment in each round, in Experimental Currency Units (ECUs), depends on a randomly drawn state and on choices both players will make that dictate an action for each of those states. This action is decided in part by the Writer, who moves first, and in part by the Receiver, who moves second after observing the Writer's choices. The details of this process will be described below.

States: There are 5 random states that can occur, numbered 1, 3, 5, 7, and 9. The states that occur in this experiment will be computer generated and all states will be equally likely in each round. There will be a state drawn after the Writer has written the Writer's rule.

Writer's Rule: In this experiment, the Writer will be writing a rule. This rule will indicate a 'low state,' a 'high state,' and a 'rule action.' The 'low state' can be any state (1, 3, 5, 7, or 9). The 'high state' can be any state (1, 3, 5, 7, or 9) that is higher than or equal to the 'low state.' The 'rule action' can be any action from 1 to 12 that is a multiple of .25. This rule will help to determine the action that is taken. If a state is drawn that is between 'low state' and 'high state', or equal to either of these states, the rule will dictate that the 'rule action' is taken. The Writer can also choose not to write a rule. The rule that the Writer writes is shown to the Receiver.

State Draw and Message Sending: After the Writer writes his/her rule, the state will be drawn. If the state that is drawn is between 'low state' and 'high state' or equal to either of those states, the 'rule action' is taken and the round will end. If a state is drawn that is below 'low state' or above 'high state,' or no rule was written, only the Writer will observe the state. The Writer, after observing the state, must send a message to the Receiver, who will see the message and then take a ‘Receiver action’. The message that the Writer can send can be one of the following numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Receiver’s Action: After the Writer writes his/her rule, the Receiver may receive a message. If the Receiver receives a message, the Receiver will observe only the message and the Writer’s rule and then take a ‘Receiver action.’ The ‘Receiver action’ can be any action between 1 and 12 that is a multiple of .25. The Receiver does not observe the state when they choose an action.

In summary, each round of the experiment will be as follows:

First: The Writer can either write a rule or not write a rule. A rule indicates three things: ‘low state’, ‘high state’, and ‘rule action.’ The ‘high state’ must be a state with number higher than or equal to the number the Writer writes down for ‘low state.’ The ‘rule action’ can be any action from 1 to 12 that is a multiple of .25.

Second: Then, after the Writer writes (or does not write) a rule, the state will be drawn. If a rule has been written and the state is between ‘low state’ and ‘high state’ or equal to ‘low state’ or ‘high state,’ the computer will take the ‘rule action’ and the round will end. Otherwise, if the state drawn is less than ‘low state’ or higher than ‘high state,’ or no rule was written, the Writer will privately observe the state and then send a message to the Receiver. The possible messages are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Third: If the Receiver receives a message, the Receiver observes the message and the rule and then takes a ‘Receiver action.’ The ‘Receiver action’ can be any action from 1 to 12 that is a multiple of .25.

At the end of each round, you will be shown the decisions of both you and your partner, the action taken, and your earnings for the round.

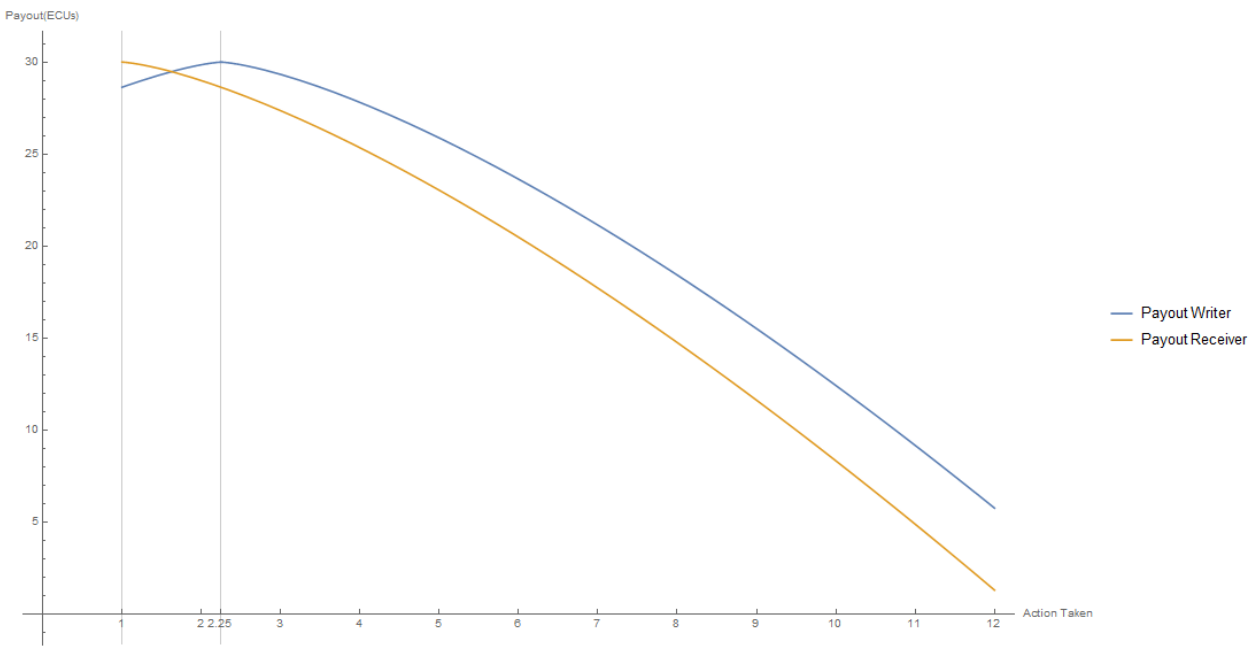
Your payout, depending on the action and state, is detailed graphically on the next page. It is decided using the following formula:

Writer’s Payout $= 30- \mid state + 1.25 - action\ taken \mid^{1.4}$.

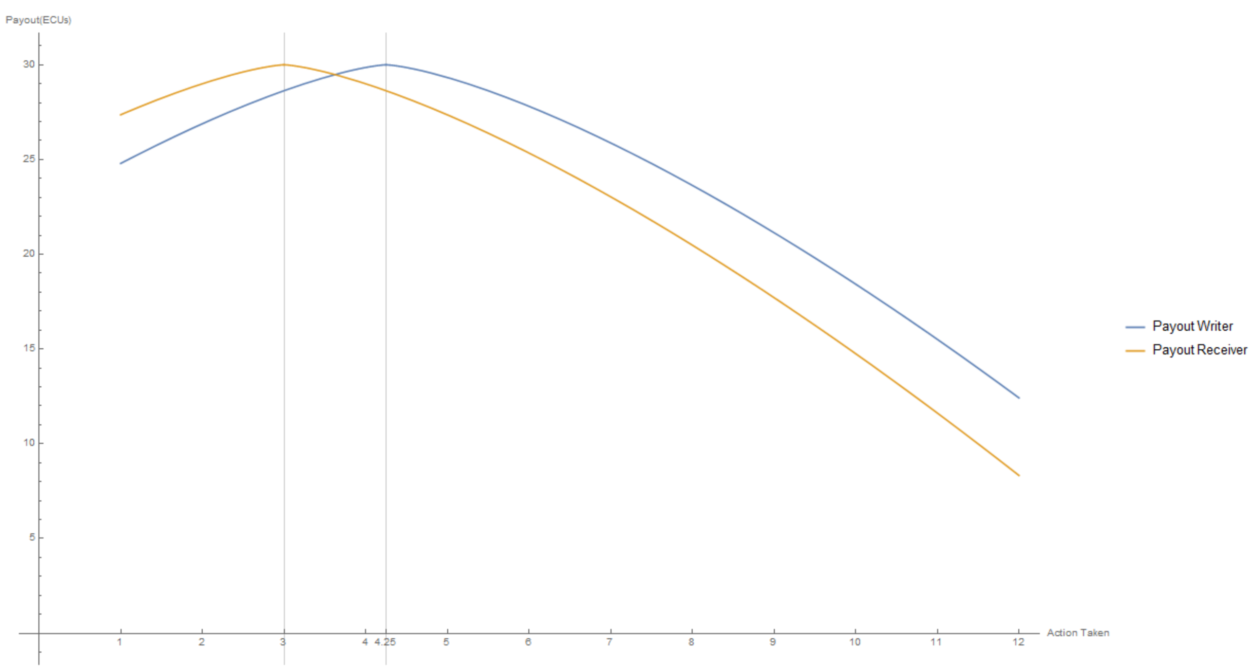
Receiver’s Payout $= 30- \mid state - action\ taken \mid^{1.4}$

Verbally, the Writer’s payout is: take the absolute difference between the state plus 1.25 and the action taken and raise that number to the power of 1.4; then subtract that number from 30. The Receiver’s Payout is: take the absolute difference between the state and the action taken and raise that number to the power of 1.4; then subtract that number from 30. This payout is displayed graphically on pages 5-7 for each state. Note that the action that gives the Writer and the Receiver the highest payout in each state is indicated by a line. You will also be able to see your payout on sliders in the experiment itself. The sliders allow you to adjust the action in each state to see possible payouts. The sliders will be on the left side of the screen at any point when you are not in a waiting screen. (Note: The sliders can lag a bit, so be careful that you have the correct number selected)

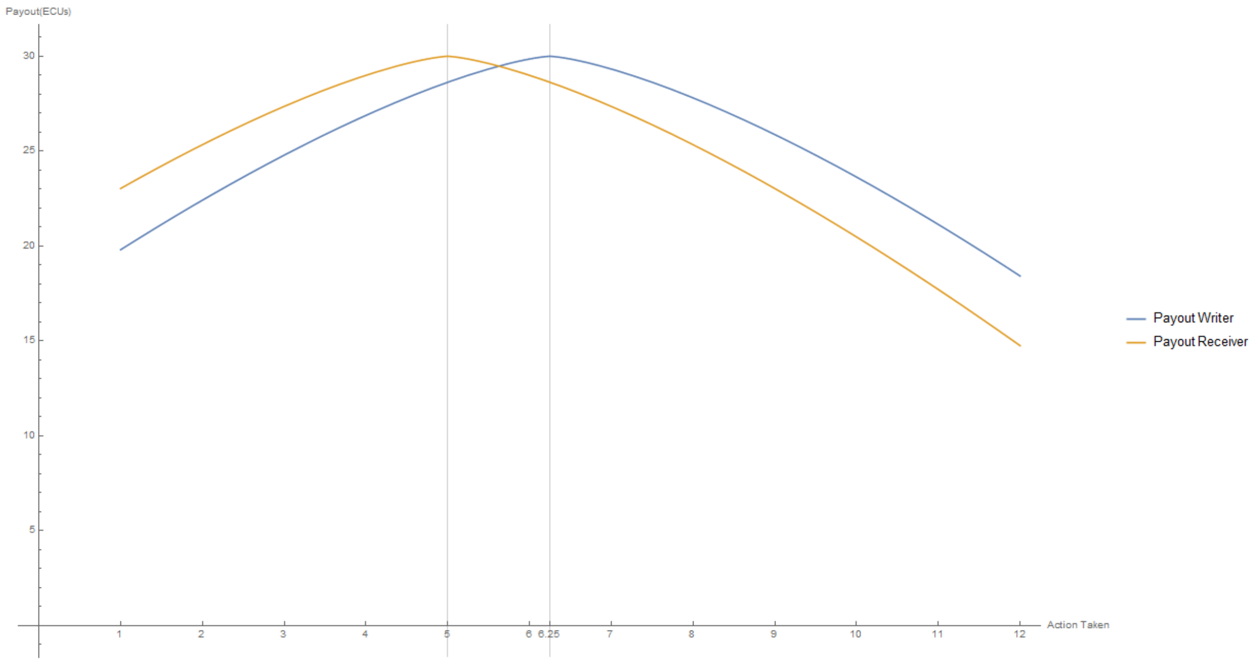
You will play 2 practice rounds of the task by yourself as both the Writer and the Receiver where you will be quizzed on the things that happen in those trials at the end of each trial period. After that, the task will be repeated 40 times, with random matching in each round, and where your role will stay fixed as either the Writer or the Receiver. Your total earnings from this experiment will be your earnings from 2 of the 40 periods, drawn randomly by you at the end of the experiment, plus your show up fee of $6. The payments in each period will be recorded in Experimental Currency Units (ECUs). Each ECU is worth 33 cents (.33 dollars).



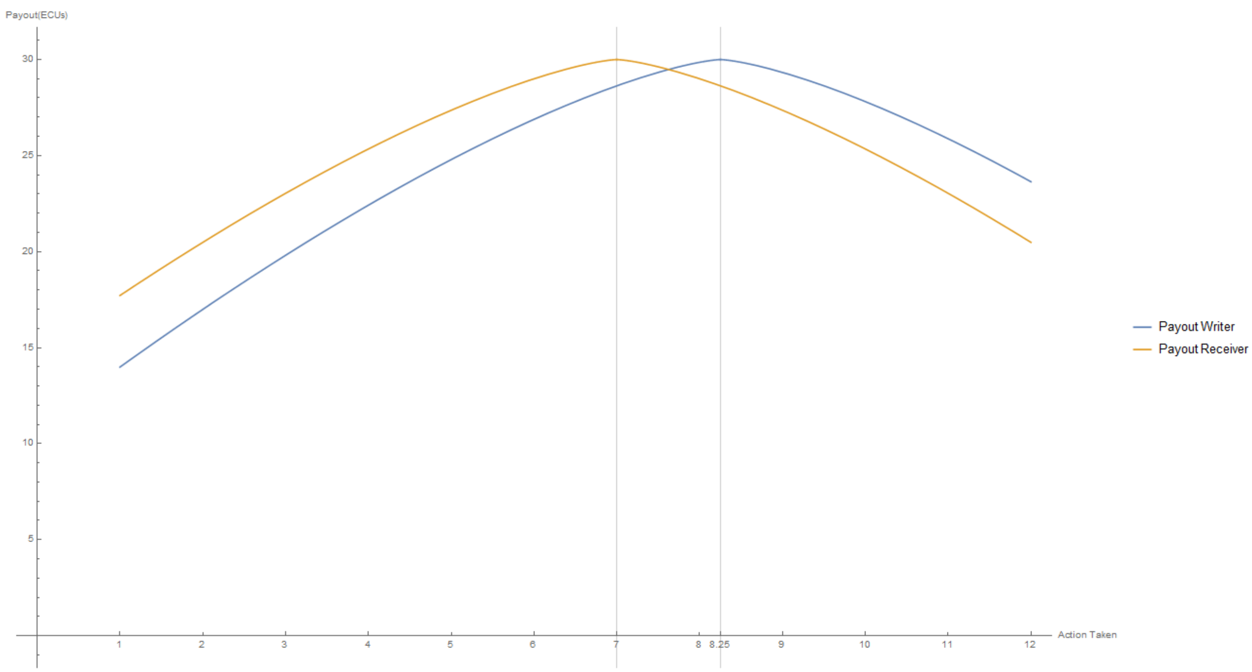
State=1 Payouts



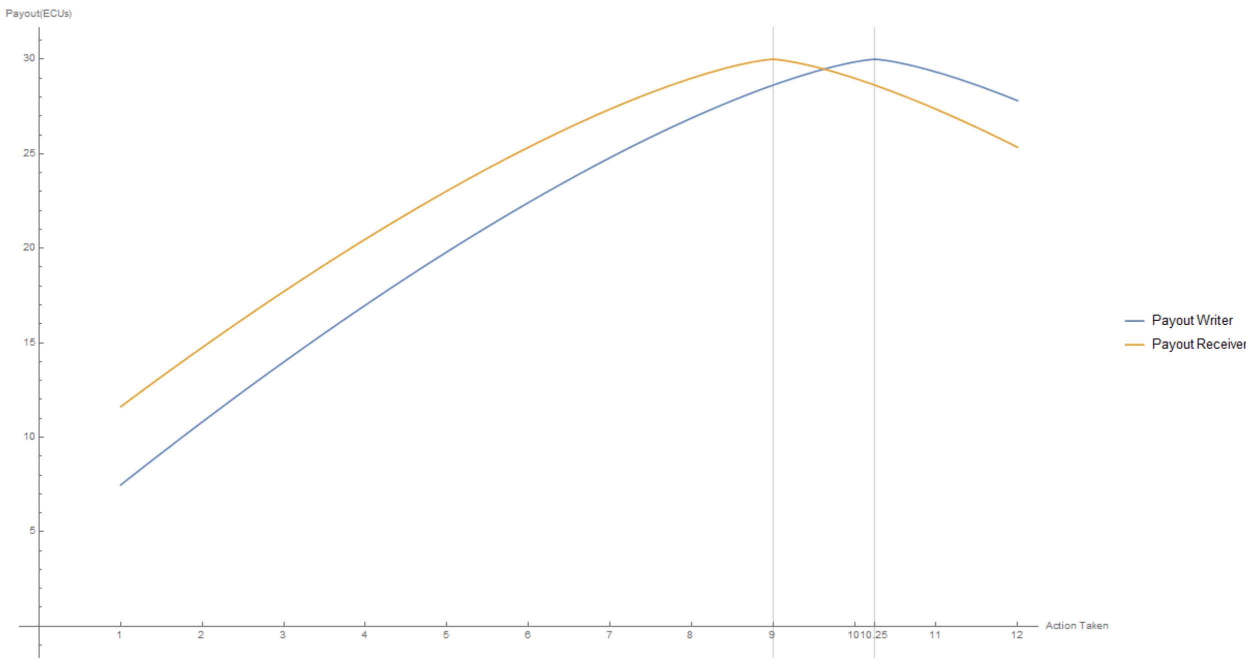
State=3 Payouts



State=5 Payouts



State=7 Payouts



State=9 Payouts